

Magnetoconductivity of low-dimensional disordered conductors at the onset of the superconducting transition

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(Received 12 March 2009; revised manuscript received 8 June 2009; published 25 June 2009)

Magnetoconductivity of the disordered two- and three-dimensional superconductors is addressed at the onset of superconducting transition. In this regime transport is dominated by the fluctuation effects and we account for the interaction corrections coming from the Cooper channel. In contrast to many previous studies we consider strong magnetic fields and various temperature regimes, which allow to resolve the existing discrepancies with the experiments. Specifically, we find saturation of the fluctuations induced magnetoconductivity for both two- and three-dimensional superconductors at already moderate magnetic fields and discuss possible dimensional crossover at the immediate vicinity of the critical temperature. The surprising observation is that closer to the transition temperature weaker magnetic field provides the saturation. It is remarkable also that interaction correction to magnetoconductivity coming from the Cooper channel, and specifically the so called Maki-Thompson contribution, remains to be important even away from the critical region.

DOI: [10.1103/PhysRevB.79.212511](https://doi.org/10.1103/PhysRevB.79.212511)

PACS number(s): 74.25.Fy, 74.40.+k, 74.78.-w

Magnetotransport measurements provide a direct access to localization effects in disordered metals and superconductors. This is a sensitive technique to probe the nature of electron coherence and specifically, magnetoresistance experiments opens a way to determine the electron dephasing time (τ_ϕ), which plays a key role in quantum-interference phenomena. Negative anomalous magnetoresistance in metals¹ is successfully explained by the weak-localization effect, see Ref. 2 for the review. Situation becomes more interesting in superconductors since in the vicinity of the critical temperature transport is dominated by the superconductive fluctuations³ and one must necessarily account for the interaction corrections coming from the Cooper channel.⁴⁻⁶ The existing theory of the magnetotransport in d -dimensional superconductors^{2,7,8} predicts that *excess* part of magnetoconductivity $\delta\sigma_d(H) = \sigma_d(H) - \sigma_d(0)$ for weak spin-orbit scattering is given by

$$\delta\sigma_d(H) = \delta\sigma_d^{\text{WL}}(H) + \delta\sigma_d^{\text{MT}}(H) = \frac{e^2}{2\pi^2\hbar} \left(\frac{eH}{\hbar c} \right)^{\frac{d}{2}-1} \times [1 - \beta(T)] Y_d(\omega_H \tau_\phi), \quad (1)$$

where $\omega_H = 4eDH/c$ is the cyclotron frequency in a disordered conductor and D is the corresponding diffusion coefficient. The dimensionality dependent universal function $Y_d(x)$ is known from the localization theory. In the two-dimensional case it is given by⁹

$$Y_2(x) = \ln x + \psi\left(\frac{1}{2} + \frac{1}{x}\right), \quad (2)$$

with the limiting cases $Y_2(x) \approx x^2/24$ for $x \ll 1$ and $Y_2(x) \approx \ln x$ for $x \gg 1$, where $\psi(x)$ is the digamma function. In the three-dimensional case¹⁰

$$Y_3(x) = \sum_{n=0}^{\infty} \left[\frac{2}{\sqrt{n+1+\frac{1}{x}} + \sqrt{n+\frac{1}{x}}} - \frac{1}{\sqrt{n+\frac{1}{2}+\frac{1}{x}}} \right], \quad (3)$$

with the limits $Y_3(x) \approx x^{3/2}/48$ for $x \ll 1$ and $Y_3(x) \approx 0.605$ for $x \gg 1$.

The first term in the square brackets of Eq. (1) corresponds to the conventional weak-localization (WL) correction.^{9,10} The second term, containing temperature dependent $\beta(T)$ factor, which is universal irrespective dimensionality, originates from the interaction corrections in the Cooper channel, and specifically from the Maki-Thompson (MT) diagram.^{5,6} It was Larkin's insightful observation⁷ that interactions with superconductive fluctuations lead to the same magnetic-field dependence of the conductivity as in the case of weak localization. Since $\beta(T)$ is strongly temperature dependent

$$\beta(T) = \frac{\pi^2}{6} \frac{1}{\ln^2(T/T_c)}, \quad \ln(T/T_c) \gg 1, \quad (4)$$

$$\beta(T) = \frac{\pi^2}{4} \frac{1}{\ln(T/T_c)}, \quad \ln(T/T_c) \ll 1, \quad (5)$$

where T_c is the critical temperature of a superconductor, $\delta\sigma_d^{\text{MT}}(H)$ dominates against $\delta\sigma_d^{\text{WL}}(H)$ in the immediate vicinity of the transition when $T - T_c \lesssim T_c$. It is worth emphasizing that MT contribution remains essential even away from the critical region as well as stays important in the nonsuperconductive materials, having repulsive interaction in the Cooper channel, which is in contrast to the Aslamazov-Larkin (AL) contribution.⁴ Furthermore, since $\beta(T) > 0$ for any sing of the interaction in the Cooper channel, Maki-Thompson correction reduces the magnetoconductivity in the absolute value.

It turns out, however, that in general Eq. (1) fails to reproduce experimental observations in both two-¹¹⁻¹⁴ and three-dimensional¹⁵⁻¹⁹ cases, except for the limit of relatively weak magnetic fields. Careful experimental analysis revealed that the discrepancy stems from the Maki-Thompson part of

Eq. (1), which ceases to follow $\delta\sigma_d^{\text{MT}}(H) = -\beta(T)\delta\sigma_d^{\text{WL}}(H)$ above the certain magnetic field. Strictly speaking, validity of Eq. (1) relies essentially on the assumptions

$$\omega_H \lesssim T \ln \frac{T}{T_c} \lesssim T, \quad \tau_\phi^{-1} \lesssim T \ln \frac{T}{T_c}, \quad (6)$$

which set a lower bound for its applicability in the magnetic field, so that this discrepancy is not surprising and should be anticipated. In application to the two-dimensional case this problem and subsequent generalizations were realized in Refs. 20–23, for the three-dimensional case there is no theoretical formulation, with the noticeable exceptions,^{24,25} where layered superconductors were considered.

The purpose of the present work is to give a unified and complete theory of the magnetotransport in the fluctuating regime of superconductors. We relax on the assumptions of Eq. (6) and treat the regime of strong magnetic field for both two- and three-dimensional cases. The essential results can be summarized as follows: (i) the excess part of fluctuation-induced magnetoconductivity, including both Maki-Thompson and Aslamazov-Larkin contributions, saturates to its negative, zero-field values at already moderate magnetic fields $T \ln \frac{T}{T_c} \lesssim \omega_H \lesssim T$ for any dimensionality. This fact has clear physical explanation and is supported by all available experiments. Indeed, magnetic field can be thought as an effective *depairing factor*, which shifts critical temperature driving the system away from the transition, thus suppressing fluctuation effects. At the technical level this happens because magnetic field enters as the mass of the fluctuation propagator. (ii) Maki-Thompson magnetoconductivity contribution dominates against Aslamazov-Larkin for the most of the experimentally accessible temperatures, except for the immediate vicinity of the critical temperature. (iii) A surprising and rather counterintuitive observation is that closer one is to the transition temperature *weaker* magnetic field leads to magnetoconductivity saturation, since it is controlled by the ratio ω_H/T and not by ω_H itself. This fact is also supported by the experiments.^{17,18} Naively one would expect a completely different scenario, since proximity to a transition enhances the lifetime for fluctuating Cooper pairs and thus, stronger field is required to destroy them. (iv) As temperature is lowered one may observe a *dimensional crossover* from three-dimensional case to two dimensional when Ginzburg-Landau length (ℓ_{GL}) exceeds the thickness (b) of the film, namely when $\ell_{\text{GL}} \sim \sqrt{\frac{D}{T-T_c}} \gtrsim b$. The indication for this possibility is already seen in the experimental results of Ref. 18. Another possibility is the crossover between MT and AL contributions which in principle can be realized for thicker films or in the layered superconductors due to their highly anisotropic properties.^{24,25}

Quantitatively we find for the Maki-Thompson magnetoconductivity in the three-dimensional case (hereafter $\hbar=c=k_B=1$)

$$\delta\sigma_3^{\text{MT}}(H) = \frac{e^2}{2\pi^2\ell_H} \mathcal{B}(T) [\mathcal{Y}_3(\omega_H\tau_{\text{GL}}) - \mathcal{Y}_3(\omega_H\tau_\phi)], \quad (7)$$

where magnetic length $\ell_H = \sqrt{D/\omega_H}$ and Ginzburg-Landau time $\tau_{\text{GL}}^{-1} = \frac{8T}{\pi} \ln \frac{T}{T_c}$ were introduced. The universal scaling function reads as

$$\mathcal{Y}_3(x) = \int_0^{+\infty} \frac{dt}{\sqrt{t}} \left[\psi\left(\frac{1}{2} + t + \frac{1}{x}\right) - \ln\left(t + \frac{1}{x}\right) \right]. \quad (8)$$

The temperature-dependent factor is defined as $\mathcal{B}(T) = T\tau_{\text{GL}}/(1 - \tau_{\text{GL}}/\tau_\phi)$. With the help of well-known properties of the digamma function and for the experimentally relevant range of magnetic fields, one finds from Eq. (7) following limiting cases for the magnetoconductivity:

$$\delta\sigma_3^{\text{MT}}(H) = -\frac{e^2}{96\pi\ell_\phi} \frac{1 - \eta^2}{1 - \eta} (T\tau_{\text{GL}})(\omega_H\tau_\phi)^2, \quad \omega_H \lesssim \tau_\phi^{-1}, \quad (9)$$

$$\delta\sigma_3^{\text{MT}}(H) = -\frac{e^2}{\pi^2\ell_\phi} \frac{T\tau_{\text{GL}}}{1 - \eta} (\omega_H\tau_\phi)^{\frac{1}{2}}, \quad \tau_\phi^{-1} \lesssim \omega_H \lesssim \tau_{\text{GL}}^{-1}, \quad (10)$$

$$\delta\sigma_3^{\text{MT}}(H) = -\sigma_3^{\text{MT}}(0) + \frac{(2\sqrt{2} - 1)\zeta(\frac{3}{2})e^2}{4\pi\ell_T} \sqrt{\frac{T}{\omega_H}}, \quad \omega_H \gtrsim \tau_{\text{GL}}^{-1}. \quad (11)$$

Here $\zeta(x)$ is the Riemann zeta function, $\ell_\phi = \sqrt{D\tau_\phi}$ and $\ell_T = \sqrt{D/T}$ are dephasing and thermal lengths respectively, and we introduced $\eta = \tau_{\text{GL}}/\tau_\phi$ for compactness. These asymptotes are valid as long as $\tau_\phi > \tau_{\text{GL}}$. In the opposite limit one has to interchange $\tau_\phi \rightleftharpoons \tau_{\text{GL}}$. One sees from Eqs. (9)–(11) that excess part of the magnetoconductivity goes through the series of crossovers $\delta\sigma_3^{\text{MT}}(H) \propto H^2 \rightarrow \sqrt{H} \rightarrow \text{const}$, until it saturates to its negative and magnetic-field independent value

$$\sigma_3^{\text{MT}}(0) = \frac{e^2}{\pi\ell_T} \frac{\sqrt{T\tau_{\text{GL}}}}{1 + \sqrt{\tau_{\text{GL}}/\tau_\phi}}. \quad (12)$$

It is worth emphasizing that Eq. (9), taken at $\eta \rightarrow 0$, can be recovered from Eq. (1) in the limit when $\omega_H \lesssim \tau_\phi^{-1}$, with the help of the approximate form of $Y_3(x)$ function [Eq. (3)], as it should be of course. The saturation region is *not* captured by Eq. (1), but recovered correctly [Eq. (11)] within generalized formulation of MT magnetoconductivity. To facilitate the comparison between the theory [Eq. (7)] and experiments^{15–18} we plot on the Fig. 1 the MT magnetoconductivity at different temperatures for the material parameters taken from Ref. 17. The inset plot in Fig. 1 emphasizes quadratic magnetic-field dependence of $\delta\sigma_3^{\text{MT}}(H)$ at the low-field regions [see Eq. (9)].

For the two-dimensional case magnetoconductivity is determined by the following expression^{20,22,23}

$$\delta\sigma_2^{\text{MT}}(H) = \frac{e^2}{\pi} \mathcal{B}(T) [Y_2(\omega_H\tau_{\text{GL}}) - Y_2(\omega_H\tau_\phi)], \quad (13)$$

where $Y_2(x)$ is defined by Eq. (2). For the same range of magnetic fields as in Eqs. (9)–(11) one finds from Eq. (13)

$$\delta\sigma_2^{\text{MT}}(H) = -\frac{e^2}{24\pi} \left[1 + \frac{\tau_{\text{GL}}}{\tau_\phi} \right] (T\tau_{\text{GL}})(\omega_H\tau_\phi)^2, \quad \omega_H \lesssim \tau_\phi^{-1}, \quad (14)$$

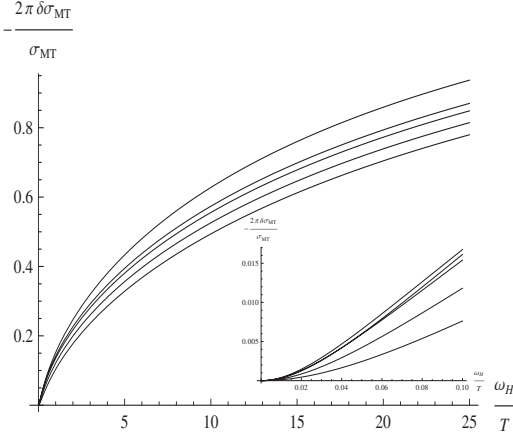


FIG. 1. Normalized Maki-Thompson magnetoconductivity for 3d case calculated at $T=1.8, 3.5, 4.4, 6.0, 8.0$ K (top to bottom) with the corresponding dephasing times $\tau_\phi=(2.24, 1.42, 1.05, 0.58, 0.28) \times 10^{-10}$ s. The material parameters are $D=6.37 \times 10^{-4}$ m²/s and $T_c=123$ mK that correspond to Ref. 17.

$$\delta\sigma_2^{\text{MT}}(H) = -\frac{e^2}{\pi} \frac{T\tau_{\text{GL}}}{1 - \frac{\tau_{\text{GL}}}{\tau_\phi}} \ln \frac{\omega_H \tau_\phi}{4e^{\gamma_E}}, \quad \tau_\phi^{-1} \lesssim \omega_H \lesssim \tau_{\text{GL}}^{-1}, \quad (15)$$

$$\delta\sigma_2^{\text{MT}}(H) = -\sigma_2^{\text{MT}}(0) + \frac{\pi e^2}{2} \frac{T}{\omega_H}, \quad \omega_H \gtrsim \tau_{\text{GL}}^{-1}, \quad (16)$$

where $\gamma_E=0.57$ is the Euler constant. Similarly to the three-dimensional case MT magnetoconductivity saturates through the series of crossovers $\delta\sigma_2^{\text{MT}}(H) \propto H^2 \rightarrow \ln H \rightarrow \text{const}$, to its field independent value determined by⁶

$$\sigma_2^{\text{MT}}(0) = \frac{e^2}{\pi} \frac{T\tau_{\text{GL}}}{1 - \tau_{\text{GL}}/\tau_\phi} \ln \frac{\tau_\phi}{\tau_{\text{GL}}}. \quad (17)$$

By comparing Eq. (9) to Eq. (14) one concludes that quadratic field dependence, $\delta\sigma_d^{\text{MT}}(H) \propto H^2$, at the lowest fields, $\omega_H \lesssim \tau_\phi^{-1}$, is apparently universal, in agreement with Eq. (1), while magnetoconductivity saturation in the two-dimensional case is stronger than in the three dimensions.

At this point we discuss the role of Aslamazov-Larkin contribution to the magnetoconductivity and compare it to $\delta\sigma_d^{\text{MT}}(H)$. In the two-dimensional case we find

$$\sigma_2^{\text{AL}}(H) = \frac{2e^2}{\pi} (T\tau_{\text{GL}}) \mathcal{H}_2(\omega_H \tau_{\text{GL}}), \quad (18)$$

$$\mathcal{H}_2(x) = \frac{1}{x} \left\{ 1 - \frac{2}{x} \left[\psi\left(1 + \frac{1}{x}\right) - \psi\left(\frac{1}{2} + \frac{1}{x}\right) \right] \right\}. \quad (19)$$

With the help of the asymptotic form of \mathcal{H}_2 function at zero field, $H \rightarrow 0$, where $\mathcal{H}_2(x \rightarrow 0) \rightarrow 1/4$, one recovers from Eq. (18) famous result⁴

$$\sigma_2^{\text{AL}}(0) = \frac{e^2}{16} \frac{1}{\ln(T/T_c)}. \quad (20)$$

Subtracting now $\sigma_2^{\text{AL}}(0)$ from Eq. (18) for the two limiting cases of low, $\mathcal{H}_2(x) \approx \frac{1}{4}(1 - \frac{x^2}{8})$, when $x \ll 1$, and high,

$\mathcal{H}_2(x) \approx 1/x$, when $x \gg 1$, magnetic fields one finds

$$\delta\sigma_2^{\text{AL}}(H) = -\frac{e^2}{16\pi} (T\tau_{\text{GL}}) (\omega_H \tau_{\text{GL}})^2, \quad \omega_H \lesssim \tau_{\text{GL}}^{-1}, \quad (21)$$

$$\delta\sigma_2^{\text{AL}}(H) = -\sigma_2^{\text{AL}}(0) + \frac{2e^2}{\pi} \frac{T}{\omega_H}, \quad \omega_H \gtrsim \tau_{\text{GL}}^{-1}, \quad (22)$$

which agrees also with the earlier results.^{24,26,27} Similarly to Eqs. (9) and (14) the low-field Aslamazov-Larkin magnetoconductivity is universal and scales quadratically with ω_H . It also saturates to the field independent value [Eq. (20)] at $\omega_H \gtrsim \tau_{\text{GL}}^{-1}$, having the same $\sim 1/H$ correction as in Eq. (16). However, if one compares the magnitude of the MT and AL contributions, for example at $\omega_H \sim \tau_{\text{GL}}^{-1}$, then it is easy to see from Eqs. (15) and (21) that $\delta\sigma_2^{\text{MT}}(H)$ dominates against $\delta\sigma_2^{\text{AL}}(H)$ by the logarithmic factor $\ln(\tau_\phi/\tau_{\text{GL}})$ and this tendency persists for the smaller fields. Although $\ln(\tau_\phi/\tau_{\text{GL}})$ depends on temperature, it actually stays practically constant, $\ln(\tau_\phi/\tau_{\text{GL}}) \sim 5$, at the experimentally addressed range of temperatures, $1 \text{ K} \lesssim T \lesssim 10 \text{ K}$ in most of the measurements, see for example Refs. 17 and 18. In the three-dimensional case expression similar to Eq. (18) can be derived,²⁴ which brings however the same conclusion about the relative importance of $\delta\sigma_3^{\text{AL}}(H)$ when compared to $\delta\sigma_3^{\text{MT}}(H)$ [Eq. (7)]. It should be emphasized that situation may be different if $\tau_{\text{GL}} > \tau_\phi$, which may happen in the layered superconductors. For this case $\delta\sigma_d^{\text{AL}}(H)$ dominates the magnetotransport in the vicinity of the critical temperature.^{24,25}

In the remaining part of the paper we outline the essential steps needed to derive Eqs. (7)–(18). Within the linear response Keldysh technique, which is proven to be very effective tool in application to the transport problems of fluctuating superconductors,^{23,28} Maki-Thompson conductivity correction is determined by the following expression

$$\sigma_d^{\text{MT}} = \frac{e^2 D}{2\pi} \sum_q \int \int_{-\infty}^{+\infty} \frac{d\epsilon d\omega}{\cosh^2 \frac{\epsilon}{2T}} \text{Im}[L^R(q, \omega)] \times |C^R(q, 2\epsilon + \omega)|^2 \left[\coth \frac{\omega}{2T} - \tanh \frac{\epsilon + \omega}{2T} \right]. \quad (23)$$

Here interaction propagator is given by $L^R(q, \omega) = [\ln \frac{T}{T_c} + \psi(\frac{Dq^2 - i\omega}{4\pi T} + \frac{1}{2}) - \psi(\frac{1}{2})]^{-1}$, while $C^R(q, \omega) = [Dq^2 - i\omega + \tau_\phi^{-1}]^{-1}$ stands for the Cooperon. In the three-dimensional case with magnetic field pointed along the z axes one has $Dq^2 \rightarrow Dq_z^2 + \omega_H(n+1/2)$ and momentum summation in Eq. (23) is performed as $\sum_q \rightarrow \frac{\omega_H}{4\pi D} \int_{-\infty}^{+\infty} \frac{dq_z}{2\pi} \sum_{n=0}^{\infty}$, where the prefactor conventionally accounts for the degeneracy in the position of Landau orbit. Passing to the dimensionless units $x = Dq_z^2/T$, $y = \omega/T$, $z = \epsilon/T$, and $w_n = \frac{\omega_H}{T}(n+1/2)$ Eq. (23) can be reduced to

$$\sigma_3^{\text{MT}}(H) = \frac{e^2}{2\pi^4 \ell_T} \frac{\omega_H}{T} \sum_{n=0}^{\infty} \int_0^{+\infty} \frac{dx}{\sqrt{x}} \int_{-\infty}^{+\infty} \frac{dzdy}{\cosh^2 \frac{z}{2}} \times \frac{y \left[\coth \frac{y}{2} - \tanh \frac{z+y}{2} \right]}{\left[\left(x + w_n + \frac{1}{T\tau_{\text{GL}}} \right)^2 + y^2 \right] \left[\left(x + w_n + \frac{1}{T\tau_{\phi}} \right)^2 + (2z+y)^2 \right]}, \quad (24)$$

where we expanded interaction propagator $L^R(q, \omega)$ at small frequencies and momenta, assuming $\max\{Dq^2, \omega\} \ll 4\pi T$. One sees from Eq. (24) that the relevant range for z integration is set by $z \sim 1$, whereas the width of the Cooperon is determined by $\max\{x, w_n, (T\tau_{\phi})^{-1}\} \ll 1$. At the end, this condition limits applicability of Eq. (7) to magnetic fields not exceeding $\omega_H \lesssim 4\pi T$, which is still sufficient to explain the magnetoconductivity saturation happening at $\omega_H \sim \tau_{\text{GL}}^{-1} \ll T$. Under this assumption one is allowed to approximate $|C^R(x+w_n, 2z+y)|^2 \approx \pi \delta(2z+y) / (x+w_n+1/T\tau_{\phi})$ in Eq. (24). Integration over z becomes immediate and gives

$$\sigma_3^{\text{MT}}(H) = \frac{e^2}{4\pi^3 \ell_T} \frac{\omega_H}{T} \sum_{n=0}^{\infty} \int_0^{+\infty} \frac{dx}{\sqrt{x} \left[x + w_n + \frac{1}{T\tau_{\phi}} \right]} \times \int_{-\infty}^{+\infty} \frac{dy}{\cosh^2 \frac{y}{4}} \frac{y \left[\coth \frac{y}{2} - \tanh \frac{y}{4} \right]}{\left[\left(x + w_n + \frac{1}{T\tau_{\text{GL}}} \right)^2 + y^2 \right]}. \quad (25)$$

The y integration can be completed with the same line of reasoning as in the z case, assuming that $\max\{x, w_n, (T\tau_{\text{GL}})^{-1}\} \ll 1$, which is consistent with the previous step, and gives a factor $2\pi / (x+w_n+1/T\tau_{\text{GL}})$. After that step summation over n is straightforward with the help of the

digamma function. Rescaling also $x \rightarrow (\omega_H/T)t$ one obtains from Eq. (25)

$$\sigma_3^{\text{MT}}(H) = \frac{e^2}{2\pi^2 \ell_H} \mathcal{B}(T) \int_0^{+\infty} \frac{dt}{\sqrt{t}} \left[\psi \left(\frac{1}{2} + t + \frac{1}{\omega_H \tau_{\text{GL}}} \right) - \psi \left(\frac{1}{2} + t + \frac{1}{\omega_H \tau_{\phi}} \right) \right]. \quad (26)$$

To recover Eq. (12) from Eq. (26) at zero magnetic field one uses following asymptote $\psi \left(\frac{1}{2} + t + \frac{1}{x} \right) \rightarrow \ln \left(t + \frac{1}{x} \right)$ valid for $x \rightarrow 0$ and an integral identity $\int_0^{\infty} \frac{dt}{\sqrt{t}} \ln \left(\frac{t+a}{t+b} \right) = 2\pi [\sqrt{a} - \sqrt{b}]$. As the final step one subtracts Eq. (12) from Eq. (26) and arrives at our major result given by Eq. (7). Corresponding calculations for the two-dimensional case [Eq. (13)] as well as derivation of Aslamazov-Larkin contribution [Eq. (18)] are completely analogous.

In conclusion we have suggested the complete theoretical description of the magnetotransport in fluctuating regime of superconductors of different dimensionality. Interaction corrections in the Cooper channel play the dominant role and are governed by the Maki-Thompson contribution. Sufficiently strong magnetic field suppresses fluctuation effects completely and magnetoconductivity is determined then by the weak-localization effect. At the immediate vicinity of the critical temperature Aslamazov-Larkin correction may become more important and one may observe MT \rightarrow AL or dimensional crossovers. These theoretical results are in good agreement with the experimental observations.¹⁵⁻¹⁸

This research was supported in part by the National Science Foundation under Grants No. PHY05-51164, No. DMR-0405212, and No. DMR-0804266.

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